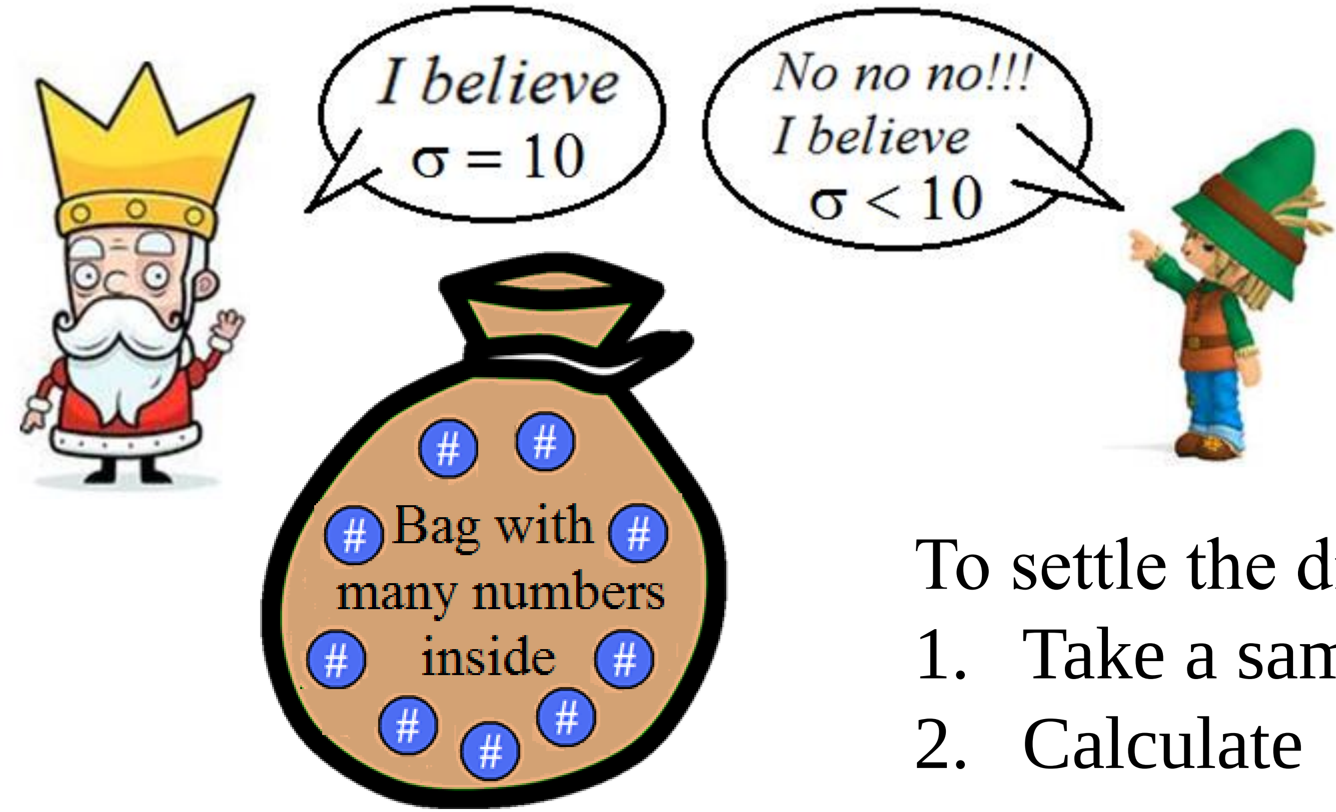
Sec. 10.4: Hypothesis Tests for a Population Standard Deviation σ (or a Population Variance σ^2)

Idea Behind Hypothesis Tests (for σ)



To settle the dispute…

1. Take a sample (only one)
2. Calculate s

If $s = 10$, believe the king

If $s > 10$, believe the king

If $s < 10$, it depends!

If $s < 10$ but close to 10, still believe the king

If $s < 10$ but far from 10, then believe the peasant

Hypothesis Tests for σ (or σ^2) Formulas and Info

Quantity you are performing a hypothesis test for: σ (or σ^2)

Significance level: $\propto$ (helps you determine the cutoff of the rejection region)

Probability distribution: χ^2 - distribution

Degrees of freedom: $df = n - 1$

Test statistic formula: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

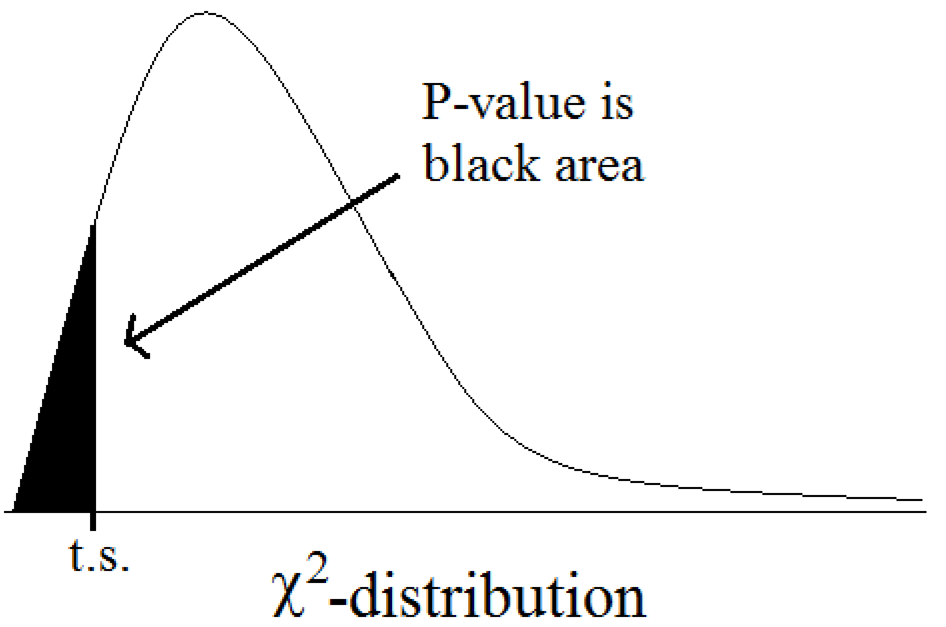
Condition: Population from which samples are drawn have a NORMAL distribution

Calculating P – Values
(for 1 population σ or σ^2 problems)

<u>For a left-tailed test</u>

The P-value is $P(\chi^2 < t.s.)$

i.e. the P-value is the area to the left of the test statistic

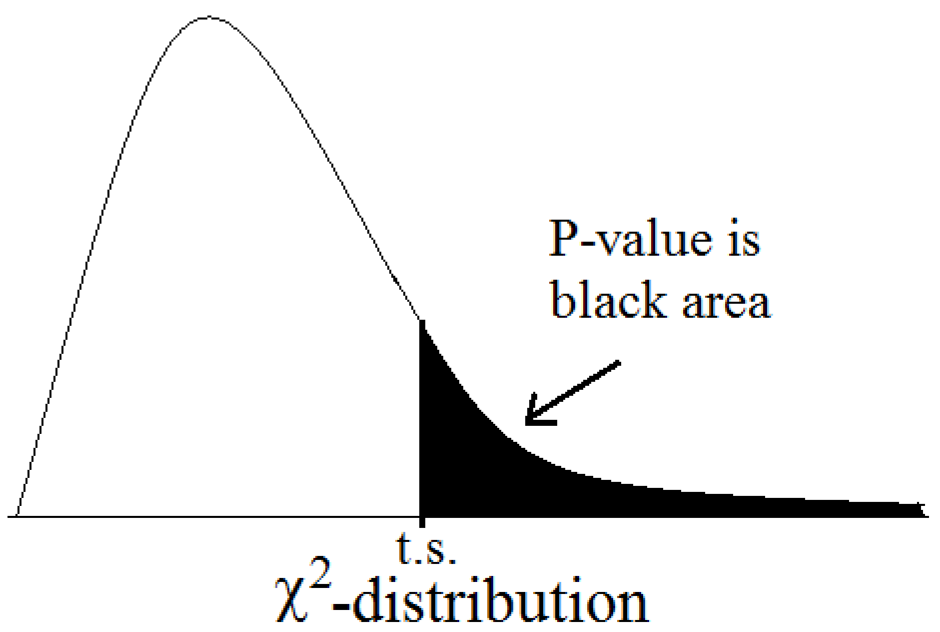


Calculating P – Values (for 1 population σ or σ^2 problems)

For a right-tailed test

The P-value is $P(\chi^2 > t.s.)$

i.e. the P-value is the area to the right of the test statistic



Calculating P – Values
(for 1 population σ or σ^2 problems)

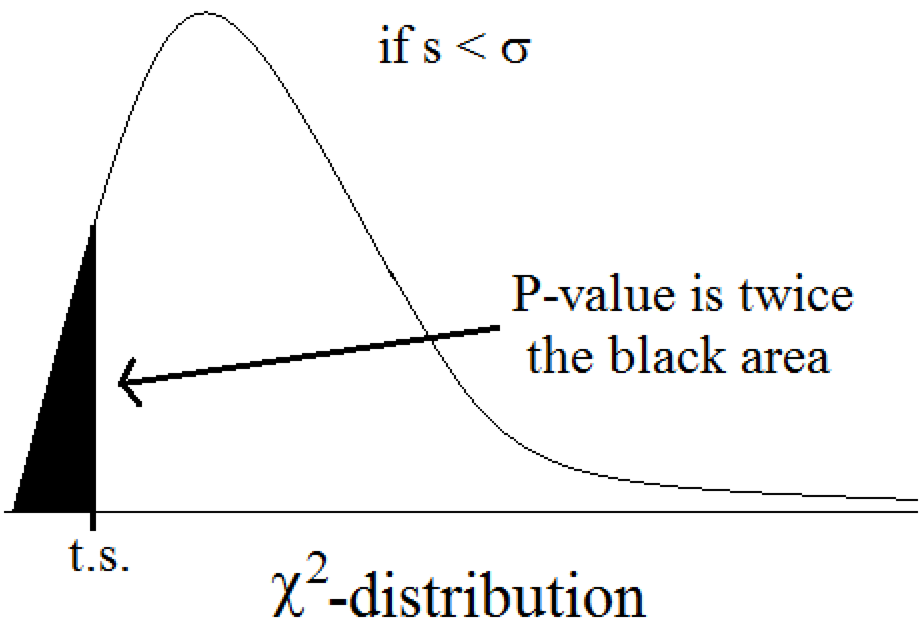
<u>For a two-tailed test</u>

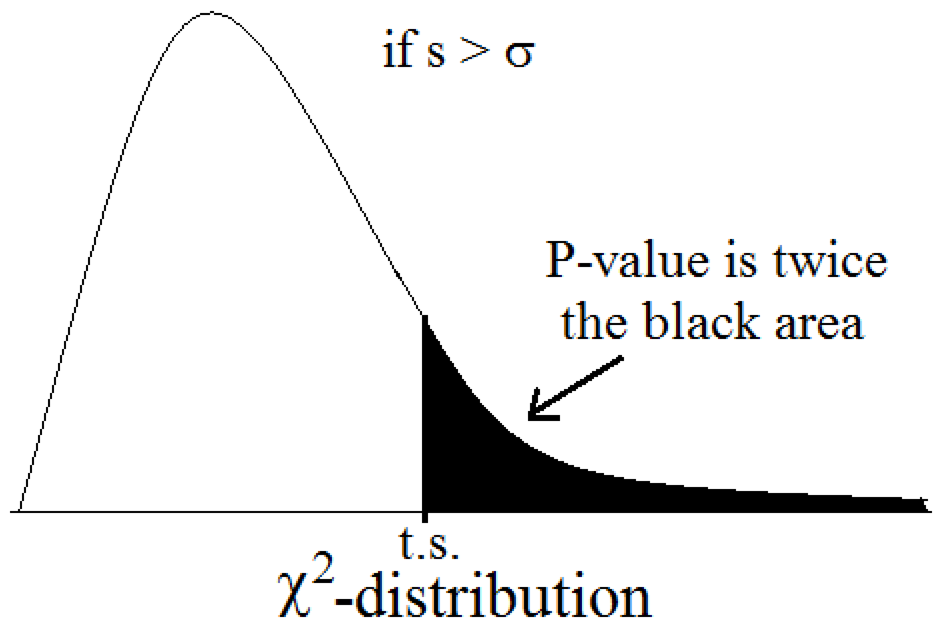
If $s < \sigma$, the P-value is $2P(\chi^2 < t.s.)$

i.e. the P-value is twice the area to the left of the test statistic

If $s > \sigma$, the P-value is $2P(\chi^2 > t.s.)$

i.e. the P-value is twice the area to the right of the test statistic





Ex 1 (Sec. 10.4 Book Example 1): The "fun size" Snickers bar is supposed to weigh 20 grams. Because the penalty for selling bars under their advertised weight is severe, the manufacturer calibrates the machine so the mean weight is 20.1 grams. Suppose that the standard deviation of the weight of the candy was 0.75 grams before recalibration. The engineer wants to know if the recalibration results in more consistent weights. Conduct the appropriate test at the $\alpha = 0.05$ level of significance.

Weights of fun-sized snickers bars (ounces)					
19.68	20.66	19.56	19.98	20.65	19.61
20.55	20.36	21.02	21.5	19.74	

a) Use the P-value method
b) Use the rejection region method
c) What does the $\alpha = 0.05$ significance level mean in a hypothesis test?

Ex 2 (Sec. 10.4 Hw #12 pg. 512): **Counting Carbs** The manufacturer of processed deli meats reports that the standard deviation of the number of carbohydrates in its smoked turkey breast is 0.5 gram per 2-ounce serving. A dietician does not believe the manufacturer and randomly selects eighteen 2-ounce servings of the smoked turkey breast and determines the number of carbohydrates per serving. The standard deviation of the number of carbs is computed to be 0.62 gram per serving. Is there sufficient evidence to indicate that the standard deviation is not 0.5 gram per serving at the $\alpha = 0.05$ significance level?

a) Use the P-value method

b) Use the rejection region method

Ex 3 (Sec. 10.4 Hw #14 pg. 512): **Filling Bottles** A certain brand of apple juice is supposed to have 64 ounces of juice. A quality control manager would like to know if the machines that fill these bottles of juice are calibrated correctly. One aspect of calibration is the variability in the amount of juice in the bottle. Suppose the machine is calibrated so than it fills the bottles with a standard deviation of 0.04 ounces. To test this, a quality control manager selects 22 bottles of apple juice and measures its contents. The data is given below.

Ounces of apple juice					
64.05	64.05	64.03	63.97	63.95	64.02
64.01	63.99	64.00	64.01	64.06	63.94
63.98	64.05	63.95	64.01	64.08	64.01
63.95	63.97	64.10	63.98		

Ex 3 (Sec. 10.4 Hw #14 pg. 512): **Filling Bottles**

Do the sample data suggest the machine may be "out of control," that is, have too much variability? Use the $\alpha = 0.05$ level of significance.

a) Use the P-value method

b) Use the rejection region method